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Robust Optimization of Portfolio's Value at Risk (VaR) by Using Random Uncertainty Series

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Abstract

This study is conducted with the aim of robust optimization of value at risk (VaR) of portfolio by using random interval uncertainty series. Therefore, the present study is an applied study in terms of research method. Statistical population of this study is consisted of companies active in Tehran Stock Exchange during the time frame of 2005 to 2014, which have been selected by using Elimination method. Results of feedback test and calculation of value at risk related to portfolios during the examined time frame indicated that calculated value at risk by using daily returns is correct. The Genetic Algorithm used in this study is a Single-step algorithm and the used selection technique is Roulette wheel with 2000 generations and each generation is consisted of 20 individuals. Results indicated the existence of algorithm convergence. Also, testing algorithm's stability indicated a small and insignificant difference between answers resulting from algorithm's Iteration in multiple times. Results also indicated that application of robust optimization method improves portfolio performance in Real-world problems with uncertainty and that internal random series have a higher level of ability in modeling asymmetric uncertainties in financial fields.

Keywords: Robust optimization, value at risk and random internal uncertainty series.

Introduction

The speed of achieving economic growth and creating motivation for investment will increase in a country which it possess reliable and active capital markets. Existence of active exchange markets has always made multiple investors to prowl and has quickened the movement of capital flow and financial resources to Productive sectors (Kianmehr & Namazi, 2007). The aim of investors is to maximize return and wealth, although for the purpose of maximizing return they also seek to reduce risk as well. Obtaining return in the process of investment is a driving force which creates motivation and is a reward for investors. Return resulting from investors is of special importance because, all investment activities are performed with the aim of gaining returns and profit (Tehrani & Noorbakhsh, Investment Management, 2009).

National economy of every country is related to its capital market performance and gets influence from it to a great extent. Optimization of portfolio refers to selection of the best combination of financial assets in a way to create maximum possible investment return and minimum possible risk. However, one of the most important challenges is the selection of appropriate and proper risk criteria. So far, so many various risk criterion such as variance (Markowitz, 1959), value at risk (VaR) (Lisimer & Pearson, 1996), conditional value at risk (CVaR) (Rockafellar & Uryasev, 2000) have been presented. Among the presented risk criterion for optimization of portfolio, VaR for practical applications have the highest level of acceptance.

However, there is one major problem about VaR which is related to the fact that this method requires to have access to a large volume of data which in this study includes means and covariance matrix. However, the above data often are prone to error and are contaminated with noise. Portfolio optimization based on inaccurate point estimations might be strongly misleading and VaR might not be able to provide the expected efficiency and might be way worse than the calculated optimized value for it. In practice, it can be way worse than the optimized calculated value. Hence, it is necessary to apply a method which accounts for uncertainties of the existing risks in the process of selecting portfolio.

Sajadi et al. (2012) have presented a method for robust portfolio optimization with considering Cardinality constraints. Uncertainty in average expected return rate of capitals was considered in a symmetrical uniform distribution which in fact is equal to uncertainty hypercube series. Considering the non-linearity of cardinality constraints, genetic algorithm is used of problem optimization. Although, both of the above mentioned types of uncertainty series are used frequently, there are two main problems related to them. First, data collection for accurate determination of boundaries and borders of uncertain “unknown but limited” series is very difficult in practice. Sometimes only the probable distribution of these boundaries and limits is obtainable from past (historical) data. In this case, uncertainty “unknown but limited” series practically have boundaries which are accompanied by change and fluctuation. Second, it is necessary to mention that the assumption indicating to Symmetry of Distribution in so many of applications , especially in modeling financial systems that in most of them form before we know that distributions are asymmetrical and hence, symmetrical assumption is equal to a restricting assumption, is so much restricting.

Gorkaz et al. (2010), have conducted a study with the title of “portfolio selection and optimization by using genetic algorithm based on different definitions of risk”. Genetic algorithm is one of the types of Heuristic algorithms which can successfully perform portfolio optimization problem by considering various level of risk. In the process of designing these algorithms two base models of Markowitz mean-variance and mean-semi variance were considered. For increasing efficiency and bringing the model more near to reality, some real world limitations were added to the designed model. Results of this study indicated that the developed genetic algorithm in different iteration has a high level of optimality and stability. Considering the obtained results it is concluded that there is no significant different in application of these two models (mean-variable and mean-semi variance models). By using the developed genetic algorithms, investors will be able to select an optimized portfolio.

Amiri and Khalozadeh (2006) have conducted a study with the title of “determining optimized portfolio in Iran Stock Exchange based on the theory….” and have found that so many studies in recent years have been conducted for developing risk management methods based on VaR theory. By using Genetic Algorithm (GA) an optimized portfolio can be obtained with a certain level of risk and a maximum level of profit, in addition to the fact that this portfolio has a restriction on portfolio’s risk. VaR has been also considered as risk estimation criterion. GA method is among numerical optimization algorithms which have been inspired from natural genetic and nature and evaluation of nature. The main advantage of these algorithms is their high level of flexibility in dealing with complex problem and not needing special mathematical conditions such as function’s continuity and derivation. The obtained results indicate the efficiency of the modeling method of market risk on the basis of value at risk theory and optimization method of genetic algorithms in obtaining optimized weight for the existing stock in portfolio with considering restriction on risk.

Shahrestani et al. (2010) have conducted a study with the title of “extending Markowitz theory in portfolio optimization”. This study indicates that this model in determining the share of each stock and the weight of each stock in portfolio is based on optimized stock selection for maximizing expected income of the portfolio. On one hand, this model enters mathematical expectation of the value of each stock in the model. On the other hand, this covariance model considers value fluctuations of stock as fix and Exogenous. Therefore, by combining Markowitz and sharp theories and recommendation of a new model, a more comprehensive model is present which is more efficient comparing to traditional boundaries of Markowitz. In other words, through making covariance of return rates related to stock in the selected portfolio endogenous in the developed model of Markowitz, expected return of the recommended model always at any certain level of risk would be larger or equal with expected return of Markowitz traditional model. In the proposed model, the share of the part of non-systematic risk which does not account for any reward for it as per sharp’s market theory, at each level of risk of the portfolio is always at the lowest possible level. The superiority of the proposed theory is confirmed both theoretically and experimentally by finding optimized portfolio of large Cement companies active in Tehran Stock Exchange comparing to Markowitz theory.

Shahmoradi and Zanganeh (2007) have conducted a study with the title of “calculation of value at risk for major indexes of Tehran Stock Exchange by using parametric method. In this study, by using for models of GARCH type, value at risk for 5 major indexes in Tehran Stock Exchange has been estimated in which Conditional Heteroskedasticity is observed. Considering that it is confirmed that probability distribution sequence data (which is an apparent characteristics of financial data) is wide for examined indexes, hence, t-distribution assumption models are as well estimated. Results indicate that this group of models explains mean behavior and data variance in a good way and that assuming t-distribution does not create any improvement in estimations results. In estimating value at risk, obtained results indicated the importance of paying attention to wide distribution sequence of data; in addition, this Risk assessment model has less sensitivity to the type of probability distribution function. In general, indexes of price and cash return, industry and 50 most active companies have less value at risk comparing to other indexes.

Lin and Jen in a study have used a two –stage genetic algorithm for solving multi-purpose portfolio optimization problem. With consideration of Markowitz model as the base mathematical model, they have sought to maximize return and minimize investment risk. In this study, after maximizing risk and minimizing return, have sought to allocate weight to the intended stock, so that they can consider the relative importance of various goals in portfolio. Operators used in this study are intersection of a cut-off point, Additional mutation and Roulette wheel selection operators. Results of this study indicated the validity and efficiency of the relevant algorithm in optimization of portfolio.

In the past years, researchers have considered the created uncertainty due to estimation errors directly in the process of portfolio optimization. This is done in the framework of robust optimization method introduced by Ben-Tal & Nemirovski (1998) and El Ghaoui and Lebret (1997). In this method, model's inputs are not the same as conventional and normal inputs such as mathematical expectation of return rate and covariance, but are uncertainty series (Ben-Tal & Nemirovski, 2000). Two groups of uncertainty series have been frequently studied by researchers which are summarized as follows:

A) Uncertainty "unknown but limited" series such as Hypercube uncertainty and elliptical uncertainty (Faboozi et al., 2007).

B) Uncertainty "random symmetrical" uncertainty in which variables are random and independent and have uniform distribution.

El Ghaoui et al. (2003), have used robust optimization for presenting a framework for optimization of the worst case of VaR based on information related to The first and second torque distribution rate of return. Zhu and Fukushima (2009), have studied worst case of CVaR under hypercube and elliptic uncertainties. Chen and Tan (2009) have presented a new type of uncertainty series known as interval random uncertainty and have used it for robust optimization model of portfolio variance average model.

In this study, we are dealing with robust optimization of the worst case of VaR of portfolio based on applying a new type of uncertainty series known as interval random series. Certain characteristics of interval random series makes them appropriate and good for reflecting upper and lower deviations in actual financial data. This type of uncertainty series accounts for distributive asymmetry of deviations and leads to better results in optimization process; especially in financial fields in which deviations are not necessarily symmetric. Hence, using interval random series as uncertainty series is logical and justifiable. In this study, these series are used for presenting uncertainty in elements of mean vector and covariance matrix. After defining sources of uncertainty and formulating optimization problem with limited probability, robust optimization of the worst case of VaR of portfolio is performed. The optimization model is solved by using an intelligent hybrid approach. In numerical simulations, based on actual data from Tehran Stock Exchange market, level of efficiency improvement and economic advantages of applying this method will be studied and evaluated and probable reduction of response efficiency against its robustness against uncertainties will be studied.

Main hypotheses

- Interval random series have a higher ability in modeling asymmetrical uncertainties in financial fields.
- Application of robust optimization method improves portfolio efficiency in real world problems with uncertainty.

Secondary hypotheses

- Exact value of portfolio model parameters is not known at the time of decision-making; however, the range of their changes can be estimated.
- Range of changes on uncertain parameters of portfolio model is not necessarily linear around its average value.

Methodology

The present study is an applied study in terms of purpose. Considering the fact that data collection is supported by bibliographical studies in this study, hence, it can be considered as a field study as well.

Variables of the study

This study is conducted based on analyses of Value at Risk (VaR) and robust optimization of portfolio by using random interval uncertainty series.

Research population and sample: statistical population of this study is consisted of companies active in Tehran Stock Exchange market during the time frame of 2005 to 2014 which are selected by using elimination method and based on the following criteria:

1. In order to make sure of the Homogeneity of the statistical sample in the examined years, companies should have been listed in Tehran Stock Exchange before 2005;
2. For increasing comparability, their fiscal year should be the end of march and during the time frame of this study, companies should not have changed their fiscal period;
3. Information related to their audited financial statements for the examined time period should be available;

Results of Descriptive statistics

In this study, for selecting a portfolio consisted of 15 stocks during 7 research period (2005 to 20140), robust optimization method is performed based on the following criteria:

- Asymmetrical robust value at risk (ARVaR)
- Symmetrical robust value at risk (SRVaR)
- Interval robust value at risk (5% - 95%) $IRVaR_{5\%-95\%}$
- Interval robust value at risk (10% - 90%) $IRVaR_{10\%-90\%}$
- Normal robust value at risk (NVaR)

Table 1: Descriptive statistics of the variables

Variables		Mean	Median	Standard deviatio	Min.	Max.
95% confidence level						
Asymmetrical robust value at risk	ARVaR	-1.4435	-1.4474	0.3227	-1.9936	-0.8955
Symmetrical robust value at risk	SRVaR	-1.3926	-1.3860	0.3293	-1.9697	-0.8143
Interval robust value at risk (5% - 95%)	IRVaR-%95 %5	-1.3669	-1.3579	0.3047	-1.9017	-0.8346
Interval robust value at risk (10% - 90%)	IRVaR-%90 %10	-1.3654	-1.3479	0.3287	-1.9589	-0.8116
Normal robust value at risk	NVaR	-1.3360	-1.3220	0.2894	-1.8506	-0.8325
99% confidence level						
Asymmetrical robust value at risk	ARVaR	-1.7322	-1.7369	0.3872	-2.3924	-1.0746
symmetrical robust value at risk	SRVaR	-1.6711	-1.6631	0.3952	-2.3636	-0.9771
Interval robust value at risk (5% - 95%)	IRVaR-%95 %5	-1.6403	-1.6295	0.3656	-2.2820	-1.0015
Interval robust value at risk (10% - 90%)	IRVaR-%90 %10	-1.6384	-1.6174	0.3944	-2.3507	-0.9739
Normal robust value at risk	NVaR	-1.6032	-1.5864	0.3472	-2.2207	-0.9991

Data collection

In this study, the presented robust optimization model is simulated by using computer in programming environment and it is evaluated and verified by using collected field data.

Testing the validity of the calculated value at risk, genetic algorithm convergence and stability of genetic algorithm answers:

Results of feedback test and calculation of value at risk related to portfolios in the examined time period indicated that calculated value at risk by using daily returns is correct. The genetic algorithm used in this study is a single-stage algorithm. The selection technique used in this study is Roulette wheel with 2000 generations and 20 populations in each generation. Results obtained from convergence study indicated that the algorithm is convergent. Another importance test regarding genetic algorithm is to test its stability, that is, whether with algorithm iteration a unique optimized and identical has been obtained or not. Results indicate to a minor and unimportant difference among the answers resulting from algorithm iteration in multiple runs. The variance obtained for algorithm answers in 20 iterations is equal to zero (0.0000051975).

Research findings

In this study, for verifying robust optimization models based on Asymmetrical Robust Value at Risk (ARVaR), Symmetrical Robust Value at Risk (SRVaR), Interval Robust Value at Risk (5%-95%), Interval Robust Value at Risk (10%-90%) and Normal Robust value at risk (NVaR) Copic probable failures ratio test was used and the results of analyses are as following:

Table 2: Results of failed compensation ratio test

Model	Confidence level	LR value	Critical distribution of Chi-Square value
Asymmetrical robust value at risk (ARVaR)	95%	6.981	4.041
Asymmetrical robust value at risk (SRVaR)	95%	8.674	4.567
Interval robust value at risk (5%-95%) $IRVaR_{5\%-95\%}$	95%	5.347	3.352
Interval robust value at risk (10%-90%) $IRVaR_{10\%-90\%}$	95%	6.902	3.761
Normal Robust value at risk (NVaR)	95%	5.327	3.981

Based on table (2), since at 95% confidence level, the calculated LR based on robust optimization for the fixe above mentioned models is larger than the critical valued obtained from chi-square distribution. At 95% confidence level, it can be claimed that research hypothesis 2 indicated that application of robust optimization method improve portfolio in real world problems with uncertainty is confirmed. For comparing the ability of the five above models which are based on robust optimization based on value at risk the number of failed ratio test was used. Obtained results indicated that models of interval robust value at risk (5%-95%) and interval robust value at risk (10%-90%) have less number of failures and more success comparing to other models; hence, research hypothesis 1 indicating that interval random series have higher ability in modeling asymmetrical uncertainties in financial fields is confirmed as well.

Table 3: Results of portfolio optimization based on interval robust value at risk criterion (5%-95%) $IRVaR_{5\%-95\%}$

Portfolio formation criteria	Portfolio No.	2005				2006				2007				2008				2009			
		weight	Objective function optimized value	Portfolio return	Variance	weight	Objective function optimized value	Portfolio return	Variance	weight	Objective function optimized value	Portfolio return	Variance	weight	Objective function optimized value	Portfolio return	Variance	weight	Objective function optimized value	Portfolio return	Variance
IRVaR 5-95	1	0.1002	0.3898	0.2846	0.2571	0.1129	0.3655	0.1472	0.2573	0.1172	0.3779	0.3868	0.2367	0.1344	0.4581	0.4908	0.2013	0.1037	0.3884	0.0724	0.2089
	2	0.1121	0.4287	0.5670	0.2597	0.1273	0.5096	0.1544	0.2494	0.1319	0.3536	0.5786	0.2481	0.1021	0.5326	0.3137	0.2323	0.1251	0.5407	0.2617	0.2381
	3	0.1094	0.4412	0.5975	0.2274	0.1046	0.5074	0.5709	0.2273	0.1076	0.3722	0.1384	0.2515	0.0987	0.5044	0.4897	0.2703	0.1402	0.5182	0.5708	0.2353
	4	0.0982	0.5159	0.3162	0.2447	0.1155	0.3726	0.4445	0.2411	0.1325	0.4861	0.1039	0.2183	0.1213	0.5383	0.5120	0.2701	**0.1097**	0.5114	0.4822	0.2372
	5	0.0986	0.5474	0.5970	0.2652	0.1226	0.4779	0.2316	0.2576	0.1278	0.4081	0.5596	0.2410	0.1263	0.3732	0.4050	0.2563	0.1271	0.4852	0.4086	0.2328
	6	0.1034	0.4416	0.4875	0.2015	0.1039	0.5709	0.1578	0.2357	0.1339	0.4952	0.4399	0.2370	0.1269	0.4748	0.4500	0.2662	0.1157	0.3485	0.5428	0.2167
	7	0.3	0.5	0.0	0.2	0.3	0.3	0.2	0.2	0.2	0.3	0.1	0.2	0.2	0.4	0.2	0.1	0.2	0.4	0.4	0.2

		781	603	666	361	132	483	382	125	490	917	983	330	904	725	246	986	785	442	978	203

2010				2011				2012				2013				2014			
weight	Objective function optimized value	Portfolio return	Variance	weight	Objective function optimized value	Portfolio return	Variance	weight	Objective function optimized value	Portfolio return	Variance	weight	Objective function optimized value	Portfolio return	Variance	weight	Objective function optimized value	Portfolio return	Variance
0.1273	0.3604	0.1061	0.2335	0.1208	0.4471	0.0435	0.2142	0.1271	0.5338	0.0338	0.1924	0.1336	0.6958	0.0282	0.1710	0.1420	0.7860	0.0194	0.1710
0.1077	0.4503	0.1324	0.2419	0.1236	0.5573	0.3205	0.2369	0.1396	0.6638	0.5055	0.2319	0.1541	0.7481	0.6143	0.2270	0.1642	0.8246	0.7280	0.2140
0.1176	0.5089	0.4492	0.2413	0.1079	0.4753	0.4920	0.2031	0.0985	0.4403	0.5370	0.1671	0.0883	0.4246	0.5840	0.1259	0.0790	0.4030	0.6374	0.0894
0.1112	0.5345	0.1598	0.1994	0.1070	0.4386	0.3836	0.2130	0.1025	0.3827	0.4917	0.2270	0.0973	0.3490	0.5456	0.2390	0.1635	0.2972	0.6040	0.2416
0.1029	0.3938	0.4138	0.2111	0.1168	0.5168	0.2360	0.2522	0.1288	0.6358	0.1960	0.2922	0.1416	0.6906	0.1480	0.3222	0.1682	0.7420	0.0928	0.3650
0.1095	0.5832	0.1886	0.2545	0.1383	0.4850	0.1407	0.2637	0.1673	0.4175	0.1127	0.2735	0.1896	0.3875	0.0877	0.2814	0.2040	0.3661	0.0345	0.2912
0.3238	0.5701	0.5377	0.2378	0.2856	0.3557	0.0598	0.2097	0.2486	0.2775	0.0428	0.1822	0.2175	0.2269	0.0376	0.1610	0.1816	0.1994	0.0260	0.1505

Table 4: Results of portfolio optimization based on interval robust value at risk criterion (10%-90%) $IRVaR_{10\%-90\%}$

Portfolio formation criteria	Portfolio No.	2005				2006				2007				2008				2009			
		weight	function optimized	Portfolio return	Variance	weight	function optimized	Portfolio return	Variance	weight	function optimized	Portfolio return	Variance	weight	function optimized	Portfolio return	Variance	weight	function optimized	Portfolio return	Variance
IR Va	1	0.120	0.564	0.567	0.236	0.114	0.354	0.114	0.227	0.111	0.359	0.432	0.24	0.1	0.4	0.3	0.2	0.1	0.4	0.0	0.237

R 10-90		4	3	7	2	8	9	6	6	0	1	7	81	241	885	514	526	327	405	743	2
	2	0.0986	0.4905	0.5418	0.2032	0.1409	0.3769	0.5255	0.2448	0.1404	0.3844	0.2401	0.2128	0.1108	0.4736	0.4884	0.2493	0.1204	0.3593	0.4473	0.1993
	3	0.1025	0.5872	0.0643	0.2526	0.1390	0.3689	0.3612	0.2401	0.1329	0.5503	0.2496	0.2327	0.1306	0.4032	0.2121	0.2497	0.1328	0.3582	0.0498	0.2665
	4	0.1217	0.5519	0.3639	0.2085	0.1093	0.5589	0.1449	0.2132	0.1011	0.3819	0.2993	0.2406	0.1006	0.5146	0.5455	0.2197	0.1303	0.4934	0.0423	0.2591
	5	0.1016	0.5852	0.3855	0.2588	0.1070	0.4576	0.0695	0.2129	0.0976	0.4108	0.5015	0.2657	0.1277	0.5471	0.5217	0.2121	0.1249	0.5825	0.0453	0.2027
	6	0.1029	0.5415	0.2219	0.2570	0.1075	0.3582	0.0804	0.2339	0.1363	0.3945	0.3836	0.2253	0.1028	0.4707	0.2214	0.2266	0.1168	0.3984	0.4774	0.2411
	7	0.3523	0.4764	0.2173	0.2368	0.2815	0.3896	0.1711	0.2431	0.2808	0.5140	0.3130	0.2033	0.3035	0.5065	0.4340	0.2588	0.2421	0.4931	0.2330	0.2653

	2011				2012				2013				
Variance	Weight	Objective function optimized value	Portfolio return	Variance	Weight	Objective function optimized value	Portfolio return	Variance	Weight	Objective function optimized value	Portfolio return	Variance	Weight
.2575	0.1302	0.4997	0.5714	0.2468	0.1495	0.4418	0.5708	0.2368	0.1682	0.4112	0.5702	0.2270	0.1842
.2467	0.1030	0.4065	0.0557	0.2658	0.1027	0.4015	0.0420	0.2843	0.1023	0.3970	0.0295	0.3022	0.1017
.2477	0.1283	0.5165	0.2142	0.2558	0.1450	0.5187	0.2938	0.2637	0.1617	0.5330	0.3718	0.2705	0.1790
.1971	0.1317	0.5140	0.1853	0.2574	0.1457	0.6740	0.2978	0.3169	0.1587	0.8019	0.4078	0.3753	0.1709

.2563	0.1089	0.5162	0.1732	0.2218	0.1019	0.6520	0.2236	0.1878	0.0954	0.7992	0.2716	0.1528	0.0895
.2691	0.1222	0.5144	0.4113	0.2539	0.1094	0.6130	0.2751	0.2387	0.0936	0.7104	0.1413	0.2238	0.0874
.2065	0.2758	0.4473	0.2868	0.2136	0.2458	0.4198	0.1894	0.2204	0.2196	0.3938	0.0957	0.2142	0.1931

Conclusion and recommendations

Based on conducted analyses, all research hypotheses were conformed and these results are consistent with results of El Ghaoui et al. (2003), Moon & Yaawoo (2011) and Gregory et al. (2011).

Considering the results of the present study, legislating bodies including Stock Exchange as well as Tehran Stock Exchange Company, brokerage companies and investment companies should direct their attention toward the effect of value at risk in portfolios' optimization. In addition, capital market activists, decision-makers, financial analysts, potential and actual investors in stock exchange are recommended to pay special attention to relationships between value at risk and portfolios' return and optimization in analyzing investment projects in financial assets and securities because these important factors can lead to the optimized selection of investment portfolio with minimum risk and maximum profit. Also, they can increase the transparency of decision making environment and the obtained results.

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